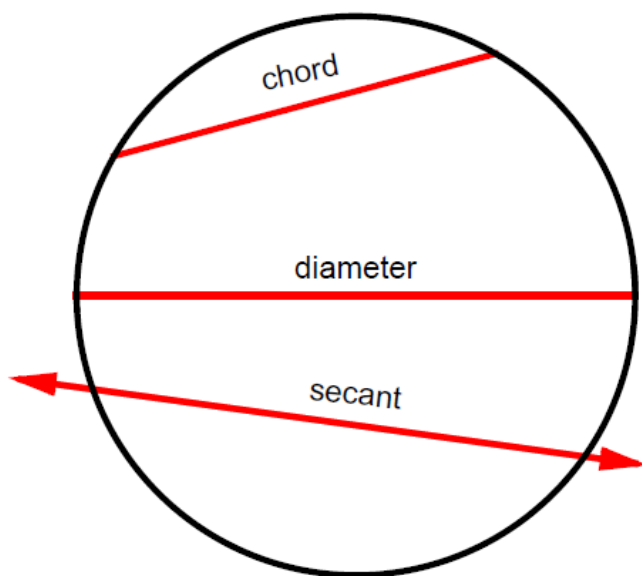


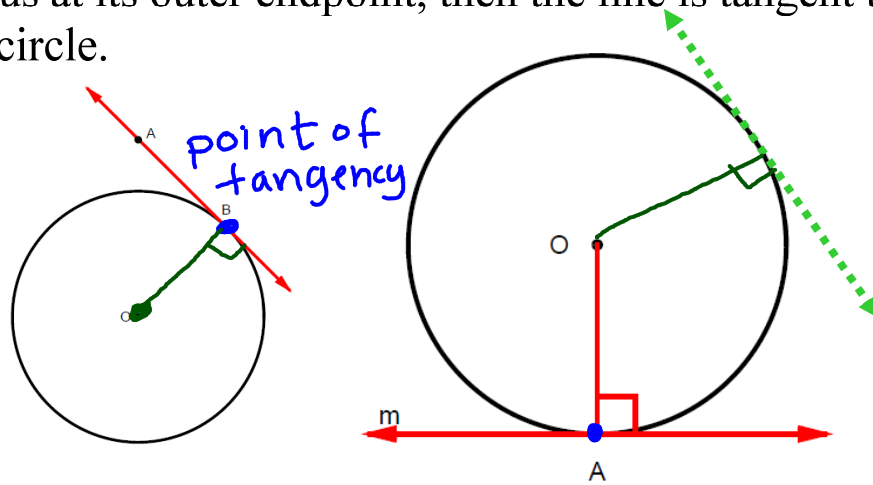
Tangents and Secants

A **secant** is a line that contains a chord.

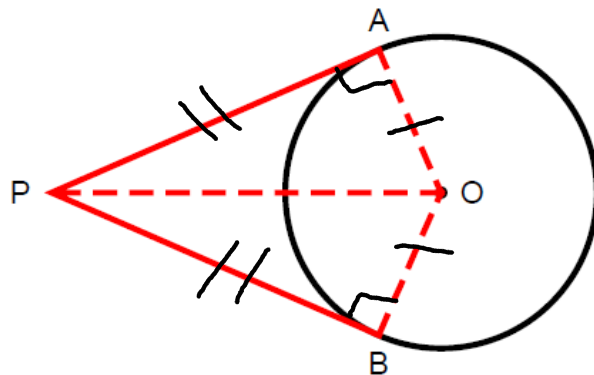


A **tangent** to a circle is a line in the plane of a circle that intersects the circle in exactly one point, called the **point of tangency**.

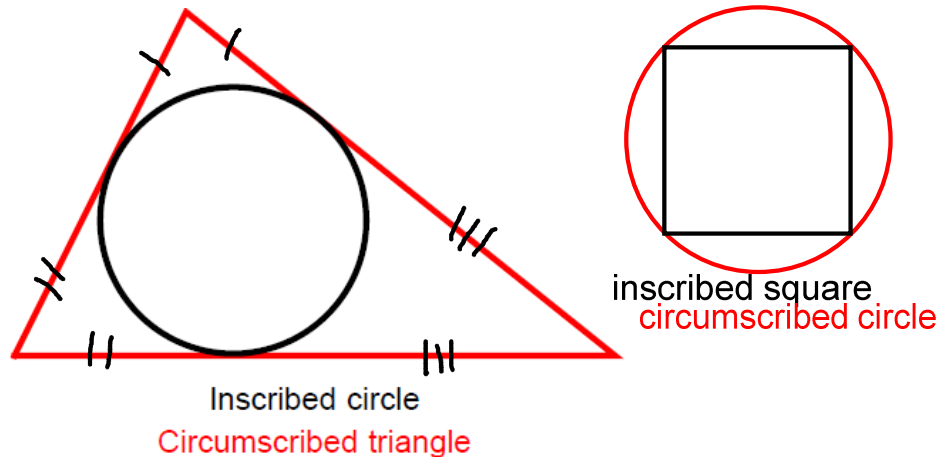
If a line in the plane of a circle is perpendicular to a radius at its outer endpoint, then the line is tangent to the circle.



Tangents to a circle from a point are congruent.

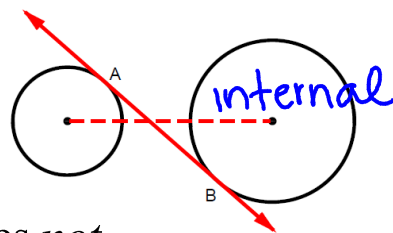


When each side of a polygon is tangent to a circle, the polygon is said to be **circumscribed about the circle** and the **circle is inscribed in the polygon**.

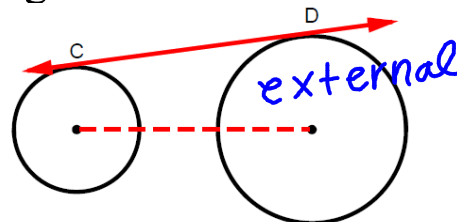


A line that is tangent to each of two coplanar circles is called a **common tangent**.

A common *internal* tangent intersects the segment joining the centers.



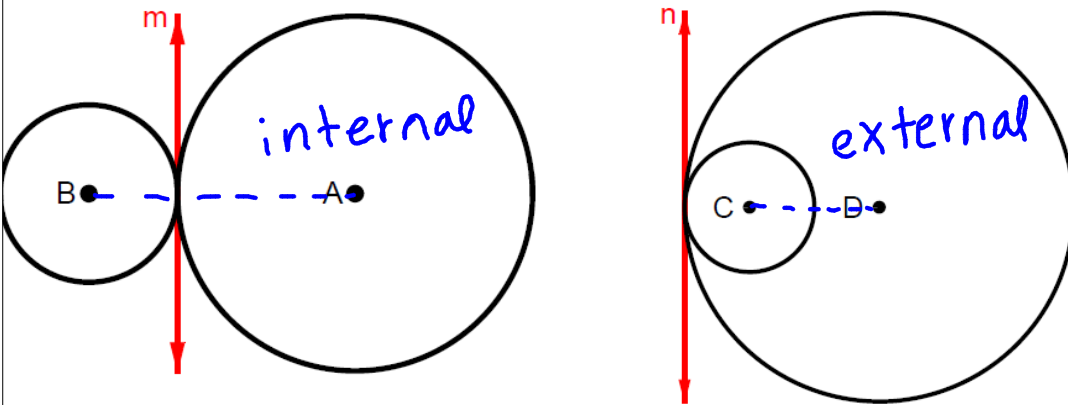
A common *external* tangent does *not* intersect the segment joining the centers.



A circle can be tangent to a line, but it can also be tangent to another circle both internally or externally. **Tangent circles** are coplanar circles that are tangent to the same line at the same point.

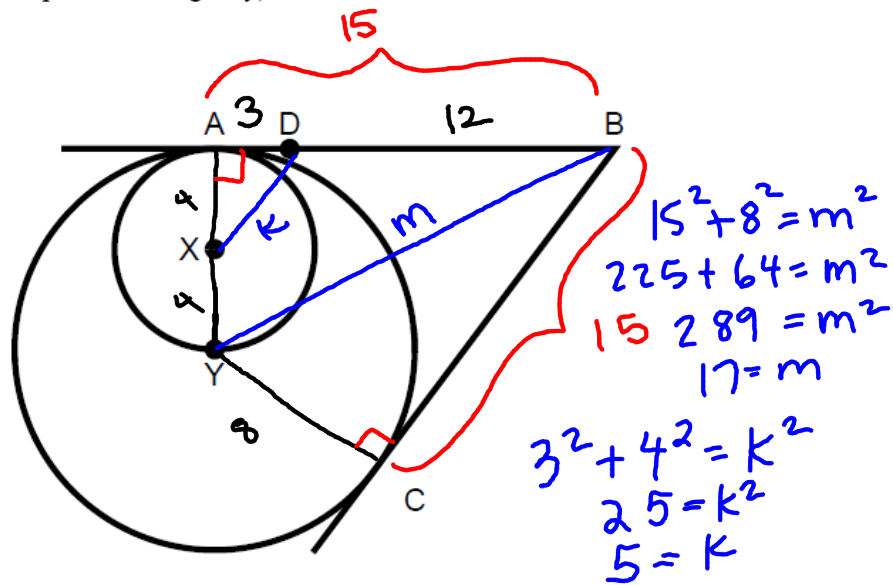
The first diagram on the left shows two circles of different sizes touching at a single point. A vertical red line with arrows at both ends, labeled 'm', is tangent to both circles at their point of contact. A dashed blue line connects the centers of the two circles, labeled 'B' and 'A' respectively. The word 'internal' is written in blue cursive between the circles.

The second diagram on the right shows two circles of different sizes touching at a single point. A vertical red line with arrows at both ends, labeled 'n', is tangent to both circles at their point of contact. A dashed blue line connects the centers of the two circles, labeled 'C' and 'D' respectively. The word 'external' is written in blue cursive between the circles.

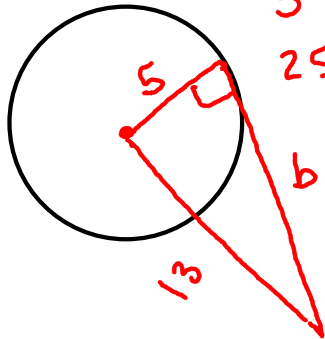


Example 1: In the diagram below, $\odot X$ and $\odot Y$ are tangent at A. \overline{AB} and \overline{BC} are tangents to $\odot Y$ (A and C are the points of tangency). $\odot Y$ has diameter 16, $AD = 3$, and $BD = 12$.

The diagram shows two circles, $\odot X$ and $\odot Y$, tangent at point A. $\odot Y$ has a vertical diameter AY of length 16. Point D lies on segment AB such that $AD = 3$ and $DB = 12$. Segment BC is tangent to $\odot Y$ at point C. Segments BD and DC are drawn. Handwritten blue annotations show calculations for m and k :

$$\begin{aligned} 15^2 + 8^2 &= m^2 \\ 225 + 64 &= m^2 \\ 289 &= m^2 \\ 17 &= m \end{aligned}$$
$$\begin{aligned} 3^2 + 4^2 &= k^2 \\ 25 &= k^2 \\ 5 &= k \end{aligned}$$


①



$$5^2 + b^2 = 13^2$$

$$25 + b^2 = 169$$

$$b^2 = 144$$

$$b = 12$$